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$$EM : O'H = MB : BH, \text{ and}$$

$$O'H : FN = AH : AN.$$

$$\therefore EM : FN = MB \times AH : AN \times BH,$$

but $MB = AM + AB = CH' + AB$, and $AN = BN + AB = CH' + AB$.

$\therefore MB = NA$, $\therefore EM : FN = AH : BH$, but $EM = AH'$, $NF = BH'$.

$\therefore AH' : BH' = AH : BH$.

$\therefore AH = AH'$, $BH = BH'$. Q. E. D.

CALCULUS.

63. Proposed by B. F. FINKEL, A. M., M. Sc., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

What is the volume removed by boring an auger hole radius r through a right cylinder radius R, the center of the auger hole to pass at a distance c from the axis of the cylinder and inclined to the axis at an angle α?

I. Solution by the PROPOSER.

Let the axis of the cylinder whose radius is R coincide with the y-axis and let the axis of the cylinder whose radius is r intersect the z-axis at a distance c from the xy-plane, being parallel to the xy-plane and making an angle α with the y-axis. Pass a plane parallel to the xy-plane through the solid common to the two cylinders and at a distance z from the origin of coördinates. The intersection of this plane with the surface of the two cylinders forms a parallelogram, whose length is $2\sqrt{R^2-z^2}$ and whose width is $2\csc\alpha\sqrt{r^2-(z-c)^2}$. Hence its area is

$$\csc\alpha\sqrt{[r^2-(z-c)^2][R^2-z^2]}.$$

$\therefore V = \int_{c-r}^{c+r} \csc\alpha\sqrt{(R^2-z^2)[r^2-(z-c)^2]}dz$. Let $y = \dfrac{p+qy}{1+y}$, p and q to be determined from the conditions that the odd powers of z in the expansion under the radical shall vanish. From this condition we find $pq = R^2$ and

$$p+q = \frac{R^2-r^2+c^2}{c} \text{ or } \frac{R^2+r^2-c^2}{r}$$

according as $c > r$ or $< r$. From these two equations we find $p =$

$$\frac{R^2-r^2+c^2+\sqrt{(R^2-r^2+c^2)^2-4R^2c^2}}{2c} \text{ or } \frac{R^2+r^2-c^2+\sqrt{(R^2+r^2-c^2)^2-4R^2r^2}}{2r}$$

and q has the conjugate value of p. Making the substitution of $(p+qy)/(1+y)$, the values of p and q being replaced by their values in terms of R, r, and c as found above. The expression for the volume reduces to the following form:

$$V=-\csc\alpha\frac{\sqrt{(R^2-r^2+c^2)-4R^2c^2}}{c}[(R^2+c^2-r^2)^2$$

$$+(R^2+c^2-r^2)\sqrt{(R^2+c^2-r^2)^2-4R^2c^2}-4R^2c^2]$$

$$[(R^2-c^2)^2-2c^2(R^2+r^2)+c^4-(R^2-c^2-r^2)\sqrt{(R^2+c^2-r^2)^2-4R^2c^2}$$

$$\times\int\Bigg[\left(1+\frac{(R^2+c^2-r^2)^2-(R^2+c^2-r^2)\sqrt{(R^2+c^2-r^2)^2-4R^2c^2}-4R^2c^2}{(R^2+c^2-r^2)^2+(R^2+c^2-r^2)\sqrt{(R^2+c^2-r^2)^2-4R^2c^2}-4R^2c^2}\,y^2\right)$$

$$\left(1+\frac{(R^2-c^2)^2-2c^2(R^2+r^2)+c^4-(R^2-c^2-r^2)\sqrt{(R^2+c^2-r^2)^2-4R^2c^2}}{(R^2-c^2)^2-2c^2(R^2+r^2)+c^4+(R^2-c^2-r^2)\sqrt{(R^2+c^2-r^2)^2-4R^2c^2}}\,y^2\right)\Bigg]^{\frac{1}{2}}$$

$$\times\frac{dy}{(1+y)^4},$$

which is of the form $k\int\sqrt{(1+my^2)(1+ny^2)}\,\frac{dy}{(1+y)^4}$. Let $y=x/\sqrt{-m}$.

Then $V=-mk\sqrt{-m}\int\sqrt{[1-x^2][1-(n/m)x^2]}\,\frac{dx}{(x+\sqrt{-m})^4}$

$$=-m\sqrt{-m}\;k\int\frac{[1-x^2][1-(n/m)x^2]dx}{[x+\sqrt{-m}]^4\sqrt{[1-x^2][1-(n/m)x^2]}}$$

$$=-m\sqrt{-m}\;k\int\frac{\{1-[(m+n)/n]x^2+(n/m)x^4\}/[x+\sqrt{-m}]^4}{\sqrt{[1-x^2][1-(n/m)x^2]}}dx.$$

By multiplying both numerator and denominator of the numerator by

$$(x^4-mx^2+m^2)-\sqrt{-m}\,x(x^2-m),$$

we reduce the numerator to a function of x^2 only. From this point by some labor we can obtain the integral in terms of logarithmic and circular, and elliptic functions of the first, second, and third kinds. We have made this integration but the result is too long and complicated to put in print.

When $c<r$, c and r change places in the formula derived by the above solution.

Corollary 1. If $c=0$, and $\alpha=\frac{1}{2}\pi$, $V=\int_0^r\sqrt{(r^2-z^2)(R^2-x^2)}dz$.

Let $z=rx$, and $r/R=e$.

Then $V=2Rr^2\int_0^1\sqrt{(1-x^2)(1-e^2x^2)}dx$.

By following out the method indicated above we get

$$V=\tfrac{8}{3}R^3[(1+e^2)E(e)-(1-e^2)F(e)].$$

Corollary 2. By Prof. Henry Heaton, Atlantic, Iowa.

If $R=r$, $V=1/\sin\alpha\int_0^{R-\frac{1}{2}c}\sqrt{R^2-(x-\tfrac{1}{2}c)^2}\,\sqrt{R^2-(x+\tfrac{1}{2}c)^2}\,dx$, where $x+\tfrac{1}{2}c$ is the distance from axis of cylinder to the plane.

Put $x=(R-\tfrac{1}{2}c)\sin\theta$, and $(R-\tfrac{1}{2}c)/(R+\tfrac{1}{2}c)=e$.

Then $V=(2/\sin\alpha)(R-\tfrac{1}{2}c)^2(R+\tfrac{1}{2}c)\int_0^{\frac{1}{2}\pi}\cos^2\theta\sqrt{1-e^2\sin^2\theta}\,d\theta$

$$=(2/\sin\alpha)(R+\tfrac{1}{2}c)^3[(1+e^2)E(e,\tfrac{1}{2}\pi)-(1-e^2)F(e,\tfrac{1}{2}\pi)].$$

Corollary 3. By Prof. G. B. M. Zerr, Lebanon, Va., and Prof. C. W. M. Black, Wilbraham, Mass.

If $R=r$ and $c=0$, $V=(16r^3)/(3\sin\alpha)$.

II. Solution by G. B. M. ZERR, A. M., Ph. D., President and Professor of Mathematics in Russell College, Lebanon, Va.

Let $x^2+z^2=R^2$, be the equation to the cylinder.

Then $(x\cos\alpha+y\sin\alpha)^2+(z-c)^2=R^2$, is the equation to the auger hole.

The limits of y are,

$$y=\frac{\sqrt{R^2-(z-c)^2}-x\cos\alpha}{\sin\alpha}\text{ and }y=-\frac{\sqrt{R^2-(z-c)^2}+x\cos\alpha}{\sin\alpha}.$$

$$\therefore\ V=(4/\sin\alpha)\int_0^R\int_{-\sqrt{R^2-x^2}}^{\sqrt{R^2-x^2}}\sqrt{R^2-(z-c)^2}\,dxdz$$

$$=(2/\sin\alpha)\int_0^R\Bigg[\left(\sqrt{R^2-x^2}-c\right)\sqrt{R^2-(\sqrt{R^2-x^2}-c)^2}$$

$$+R^2\sin^{-1}\left(\frac{\sqrt{R^2-x^2}-c}{R}\right)+\left(\sqrt{R^2-x^2}+c\right)\sqrt{R^2-(\sqrt{R^2-x^2}+c)^2}$$

$$+R^2\sin^{-1}\left(\frac{\sqrt{R^2-x^2}+c}{R}\right)\Bigg]dx.$$

This does not appear to be easy to reduce.

64. Proposed by E. S. LOOMIS, A. M., Ph. D., Professor of Mathematics in Cleveland West High School, Berea, Ohio.

Find the volume and surface generated by revolving about Y, the catenary

$$y=\tfrac{1}{2}a(e^{x/a}+e^{-x/a}),\text{ from }x=0\text{ to }x=a.$$

[*Osborne's Calculus*, page 255, example 8].